ND*nano* Summer Undergraduate Research
2016 Project Summary

1. **Student name**: SUSHRUT GHONGE

2. **Faculty mentor name**: DERVIS CAN VURAL

3. **Project title**: ESTIMATION OF QUBIT AND OTHER HAMILTONIANS

4. **Briefly describe any new skills you acquired during your summer research:** I learned about several physical aspects and mathematical techniques in quantum computing and quantum measurement problems.

5. **Briefly share a practical application/end use of your research**: Hamiltonian estimation has an important role to play in quantum computation. These techniques can be used to perform adaptive control of qubits.

Begin two-paragraph project summary here (~ one type-written page) to describe problem and project goal and your activities / results:
Coherence of quantum systems is a very important requirement for implementing quantum computation. One of the ways to maintain coherence in quantum systems is to use a technique called adaptive control. The time evolution of a quantum system occurs due to its Hamiltonian. The Hamiltonian also has some noises due to interaction with the environment which destroy coherence. If we can determine the noise, we can manually apply reverse noise so that the system stays coherent. Hence, Hamiltonian estimation finds several applications in quantum computation.

In this project, we explored two methods for Hamiltonian estimation-
1. Hamiltonian estimation by measuring several known observables-
In this method, we estimate the Hamiltonian operator in a given basis by measuring several other quantities. We assume that the experimental apparatus is able to operate any known hermitian operator on the system and that the least count and accuracy of the clock is less than the rotation period of the highest eigenvalue of the operator which is being measured. In the given basis, let α_{mn}, ($1 \leq m,n \leq N$) be the elements of the Hamiltonian matrix. The Hamiltonian is always hermitian, so $\alpha_{mn} = \alpha_{mn}^*$. Eigenvalues and eigenvectors of the Hamiltonian are functions of only α_{mn}. For $N \geq 4$ there is no analytical formula for eigenvalues and eigenvectors in terms of elements of the matrix. We choose any M pairwise non-commuting (N-dimensional) operators, $S_1, S_2, \ldots, S_M$. The elements of S_k in the given basis are S_{kmn} and the eigenvectors of S_k are v_{kj} with eigenvalues σ_{kj}, $1 \leq j \leq N$. We then write these eigenvectors in the Hamiltonian eigenvector basis. The time evolution operator which causes a state which is the superposition of these eigenvectors is given by $U(t)=e^{-iHt}$. This operator is also a function of the time t and α_{mn}. The observable is measured, then the system is allowed to evolve for some time and measured again. By measuring these observables after various time gaps, we can get sufficient linear equations in α_{mn} to determine all of them and hence the Hamiltonian.

2. Hamiltonian estimation by energy measurements

Another way to estimate the Hamiltonian is to measure the energy. If the noise has much less energy than the system itself, the eigenvalues of the Hamiltonian are almost equal to the energies measured. This will give us the Hamiltonian in its own basis, but it is not sufficient because we still do not know it in the required basis. Again we assume that it is given by α_{mn} in the required basis. When the energy is measured several times, the time period between subsequent measurements is not always exactly equal. There is an error of δt in the measurement. Hence, we expand the time evolution operator U(t+ δt) as-

$$exp(-i\hat{H}(t+\delta t)) = exp(-i\hat{H}t)\left[1 - i\hat{H}\delta t - (\hat{H})^2\frac{(\delta t)^2}{2} + i(\hat{H})^3\frac{(\delta t)^3}{6} + ...\right]$$

Here, the Hamiltonian H is the sum of the system Hamiltonian H_0 and some noise V. $H=H_0+V$, where H and V do not commute.

This can again be represented in terms of $\alpha_{mn.}$ After doing several such measurements, we can classify measurements into N^2 categories: each of the N states can transition to each of the other N-1 states or stay in the same state. We first assume all α_{mn} to be uniformly distributed. Then we use the Bayes theorem repeatedly as we observe transitions to update the probability distributions. When we get a sharp peak in the probability distribution, we claim to have an estimate of the Hamiltonian.